Estimation of Moist Air Thermodynamic Properties using Artificial Neural Network

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Abstract

In this study, the equations obtained non-iteratively are presented for moist air thermodynamic properties as a function of dry-bulb temperature and relative humidity. In this regard, an artificial neural network (ANN) was performed by using MATLAB software. In the ANN, dry-bulb temperature and relative humidity were specified as inputs, and water vapor saturation and partial pressures, wet-bulb and dew-point temperatures were determined as outputs. The sensitivity of the neural network performance was also controlled, and acceptable accuracy was obtained for all estimations for practical applications. The moist air thermodynamic properties can be alternatively estimated with the mean absolute percentage error (MAPE) of less than 0.5% by using the developed model. With respect to the acquired results, this model supplies simple and correct predictions to specify moist air thermodynamic properties non-iteratively. Determination of moist air thermodynamic properties using ANN approach is a good alternative to some other mathematical models.

Keywords: Air thermodynamic properties; Dry-bulb temperature; Wet-bulb temperature; Relative humidity; Artificial Neural Network (ANN)

Özet

Bu çalışmada, nemli havanın termodinamik özellikleri kuru termometre sıcaklığı ve bağlı nemin bir fonksiyonu olarak iterasyona gerek olmadan eşitlikler ile sunulmuştur. Bu amaçla, MATLAB programı kullanılarak yapay sinir ağları metodu uygulanmıştır. Bu metotta kuru termometre sıcaklığı ve bağlı nem girdi verisi olarak kullanılırken; su buharının doyma ve kısmi basınçları ile yaş termometre ve çiğ noktasi sıcaklıklar da çıktı olarak hesaplanmıştır. Yapay sinir ağları hassasiyeti ile beraber hesaplamalardaki doğruluklar da kontrol edilmiştir. Kullanılan model ile nemli havanın termodinamik özellikleri 0.5’ten daha düşük ortalama mutlak yüzde hata değeri ile hesaplanmıştır. Elde edilen değerlere göre bu model iterasyona gerek olmadan nemli havanın termodinamik özelliklerini belirlemede basit ve doğru tahminler

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1. INTRODUCTION

Psychrometry is related to the determination of physical and thermodynamic properties of gas-vapor mixtures. Psychrometric calculations are often necessary in a number of engineering and agricultural applications such as humidification, dehumidification, heating, ventilating, air-conditioning, meteorology, drying of food and agricultural products and grain storage. In some chemical and food industries, other systems such as air-toluene and air-benzene systems are also found [1].

The psychrometric chart shows the thermodynamic parameters of moist air at constant pressure. Knowledge of any two variables defines the state point from which all the other variables could be acquired in this chart. Even though analytical expressions have been improved for all the psychrometric parameters, calculation of the psychrometric variables cannot always be simple since some expressions are implicit in their nature. In such cases, an iterative technique should be performed in order to identify the psychrometric properties, which is very time consuming. Another popular approximation is the application of a psychrometric chart, which runs the risk of major human errors [1]. Therefore, application of the ANN approach, which is presented in this study, may be a reliable and accurate method of obtaining the output data quickly.

There are many studies using Artificial Neural Network (ANN) for the applications of air forecast and air thermodynamic properties [1-8]. In general, the psychrometric chart is used for obtaining the values of moist air thermodynamic properties. Some errors may occur when reading the parameters from the psychrometric chart. Mathematical models are developed to improve calculation accuracy. However, some equations are implicit in nature. In particular, the analytical solution of wet-bulb temperature is very difficult. On the other hand, there is no analytical solution for obtaining the wet-bulb temperature except for the trial or secant method, which is cumbersome [9]. The aim of the current study is to predict the moist air thermodynamic properties by means of a developed non-iterative method. For this purpose, an ANN model was carried out using MATLAB software. In the ANN model, relative humidity (φ) and dry-bulb temperature (T_{db}) were inputs, and wet-bulb temperature (T_{wb}), dew-point temperature (T_{dp}), partial pressure of water vapor (P_w) and water vapor saturation pressure (P_{ws}) were outputs. The developed ANN models offer various advantages over conventional deterministic analytical models including their simplicity and lack of need for iteration.

2. METHODOLOGY

2.1. Psychrometric Equations

Usually, φ and T_{db} are available from instrumental measurements. Knowing the values of φ and T_{db}, the moist air thermodynamic properties such as P_{ws}, P_w, T_{dp}, and T_{wb} can be calculated in the psychrometric chart. They can also be calculated with some known equations at various atmospheric pressures. P_{ws} at a temperature of T_{db} can be calculated by [7]:

\[ T = 273.15 + T_{db} \]  \hspace{3cm} (1)

\[ \ln(P_{ws}) = -5.6745 \times 10^3 / T - 0.5152 - 9.6778 \times 10^{-3} x T \\
+ 6.2216 \times 10^{-2} x T^2 + 2.0748 \times 10^{-3} x T^3 \\
- 9.484 \times 10^{-5} x T^4 + 4.1635 x \ln(T) \]  \hspace{3cm} (2)

for -50°C ≤ T_{db} ≤ 0°C

\[ T_{wb} \] for 0°C ≤ T_{db} ≤ 200°C
\[
\ln(P_w) = -5.8002 \times 10^2 / T - 5.5163 - 4.864 \times 10^2 x T + 4.1765 \times 10^2 x T^2 - 1.4452 \times 10^3 x T^3 + 6.546 \times \ln(T)
\]

where \( P_{\text{atm}} \) is the atmospheric pressure (kPa), \( W \) is the humidity ratio (kg/kg) and \( W_s \) is the saturation humidity ratio at \( T_{\text{wb}} \) [10].

### 2.2. Data Generation and Analysis

The monthly average values of meteorological parameters used in the study were obtained at the measuring station of Adana. Meteorological data observed between 2000 and 2009 were provided by the Turkish State Meteorological Service (TSMS). This measuring station is placed at 36°59” North latitude and 35°18” East longitude geographical coordinates. The station is situated at an altitude of 28 m above sea level and is located in the Eastern Mediterranean region of Turkey. The observed meteorological parameters are \( \phi, T_{\text{db}} \) and \( P_{\text{atm}} \). The main statistical characteristics of these variables are given in Table 1. It is shown that the mean monthly \( P_{\text{atm}} \) varies between 100.1 and 102.1 kPa with an average value of 101 kPa. \( T_{\text{db}} \) varies strongly between 6.7 and 29.8 m/s with a mean value of 19.2 °C. The mean monthly \( \phi \) is between 56.1% and 80.7%. By taking the values of \( \phi, T_{\text{db}} \) and \( P_{\text{atm}} \), the psychrometric properties such as \( P_{\text{ws}}, P_{\text{w}}, T_{\text{dp}}, \) and \( T_{\text{wb}} \) were generated using Eqs. (1-9). Table 1 also includes the important statistical properties of these variables.

| Table 1. The monthly statistical properties of observed meteorological data and moist air thermodynamic properties |
|-----------------|------|-------|-------|-------|
| Variable        | Unit | Minimum | Maximum | Mean  | Standard deviation |
| \( T_{\text{db}} \) | °C   | 6.7     | 29.8   | 19.2  | 7.3               |
| \( \phi \)      | %    | 56.1    | 80.7   | 68.9  | 5.7               |
| \( P_{\text{atm}} \) | kPa  | 100.1   | 102.1  | 101.0 | 0.48              |
| \( P_{\text{ws}} \) | kPa  | 0.98    | 4.20   | 2.43  | 1.04              |
| \( P_{\text{w}} \)  | kPa  | 0.59    | 3.29   | 1.69  | 0.78              |
| \( T_{\text{dp}} \) | °C   | -0.5    | 25.6   | 13.4  | 7.3               |
| \( T_{\text{wb}} \) | °C   | 3.7     | 26.7   | 15.6  | 6.8               |

### 2.3. Artificial Neural Networks

Artificial Neural Networks (ANNs) generate a number of very simple interconnected processing elements [11]. It can be described as a system which occurs with many nonlinear artificial neurons running in parallel, which may be created as one layer or multiple layers. In recent years there has been increased interest in ANNs. The ANN models have been applied successfully in different fields of engineering, mathematics, meteorology, medicine, neurology, psychology and economics, in adaptive and robotic control, in thermal and electrical load estimations and many other areas [12,13].
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Neuron is a primary processing element of an ANN. The network generally includes an input layer, the output layer and hidden layers [14,15]. A neuron \( j \) can be depicted mathematically with the following equations [16];

\[
\begin{align*}
\mathbf{u}_j &= \sum_{i=0}^{p} w_{ji} y_i \\
\mathbf{y}_j &= \varphi(\mathbf{u}_j - \theta_j)
\end{align*}
\]

(10) and (11)

The ANN collects a set of inputs or signals \( y \) with weight \( w \), calculates a weighted mean of them \( u \) using the summation function and then uses some activation function \( \varphi \) to generate an output \( y \). The utilizing of threshold \( \theta \) has the effect of performing an affine transformation to the output \( u \) of the linear combiner. The following equation shows the sigmoid logistic non-linear function expressed as follows [12]:

\[
\varphi(x) = \frac{1}{1 + e^{-x}}
\]

(12)

2.4. Parameters used for Error Analysis

The performances of the models were evaluated by using coefficient of determination \( (R^2) \), the mean absolute error (MAE) and the mean absolute percentage error (MAPE). MAE and MAPE are defined as [12]:

\[
\begin{align*}
MAE &= \frac{1}{n} \sum_{i=1}^{n} |p_i - m_i| \\
MAPE &= \frac{1}{n} \sum_{i=1}^{n} \left| \frac{p_i - m_i}{p_i} \right| \times 100
\end{align*}
\]

(13) and (14)

where \( n \) is the total number of data, \( p \) is the predicted value and \( m \) is the measured value.

3. RESULTS AND DISCUSSIONS

The moist air thermodynamic properties can be directly simulated as a function of the dry-bulb temperature, relative humidity, atmospheric pressure and the following non-linear equation is considered:

\[
(P_{ws}, P_w, T_{dp}, T_{db}) = f(T_{db}, \varphi, P_{atm})
\]

(15)

In the ANN method, the most important point is to choose the predictor variables that supply the best estimation equation for modeling of the dependent variable. In order to find out \( P_{ws} \), it is sufficient to know the value of the \( T_{db} \) parameter. Therefore, \( T_{db} \) is selected as the independent variable in modeling of \( P_{ws} \). As can be seen from Eq. (4), \( P_w \) value depends on the parameters of \( \varphi \) and \( P_{ws} \) (hence \( T_{db} \)). Therefore, \( \varphi \) and the derived \( P_{ws} \) equation were used in modeling of \( P_{ws} \). Furthermore, \( T_{dp} \) and \( T_{wb} \) values indirectly depend on the parameters of \( \varphi \), \( T_{db} \) and \( P_{atm} \). In particular, \( T_{wb} \) is very difficult to solve analytically. As seen in Table 1, \( P_{atm} \) the value changes between 100.1 kPa and 102.1 kPa and it is not selected as an independent variable because its value does not change too much. In addition, \( \varphi \) and \( T_{db} \) are the two most important meteorological parameters which are measured easily throughout the world including Turkey. The moist air thermodynamic properties can be calculated easily with a high level of accuracy depending on only two variables \( (\varphi \text{ and } T_{db}) \). As mentioned earlier, the purpose of this study is the prediction of thermodynamic properties of moist air by using the equations, which are uncomplicated, and do not require too many variables and iterations. For instance, \( \varphi \) and \( T_{db} \) were selected as predictor variables to acquire the predictive equation for modeling of the moist air thermodynamic properties.

A total of 120 data records were used for the modeling of moist air thermodynamic properties. This data set was divided into two parts: the training and testing data sets. The measured data between 2000 and 2007 were applied for training and the 24 months of data of 2008 and 2009 were used for testing. After different training algorithms were used, Levenberg–Marquardt (LM) learning algorithm was used in the network of the present study. The input layer does not include any transfer function. Linear transfer function (purelin)
and Logistic sigmoid transfer function (logsig) were applied in the output and hidden layers of the network. The number of hidden layers was chosen as small to avoid very complex equations which will obtained from the simulations. Therefore, the number of hidden layers was chosen as 1 to find $P_{ws}$ and as 2 in order to find $T_{dp}$ and $T_{wb}$ parameters. The models were tested by testing data set, which was not used during the training process. Finally, the moist air thermodynamic properties were modeled. For $55\% \leq \varphi \leq 80\%$ and $5^\circ C \leq T_{db} \leq 30^\circ C$, the new equations for the outputs are given with Eqs. (16-19). These equations can be used for the prediction of the water vapor saturation pressure, partial pressure of water vapor, dew-point temperature and wet-bulb temperature.

$$P_{ws} = 39.52098 - \frac{39.66352}{1 + \exp(0.06228T_{wb} - 3.95283)}$$

(16)

$$P_p = \frac{\varphi}{100} \left[ 39.52098 - \frac{39.66352}{1 + \exp(0.06228T_{wb} - 3.95283)} \right]$$

(17)

$$T_{dp} = -147.41387 - \frac{21.35108}{1 + \exp(-0.03933\varphi + 0.01954T_{wb} + 3.38298)} + \frac{352.47471}{1 + \exp(-0.00438\varphi - 0.00997T_{wb} + 0.60604)}$$

(18)

$$T_{wb} = 131.42789 + \frac{39.90177}{1 + \exp(0.01827\varphi - 0.03291T_{wb} - 1.69131)} - \frac{206.98595}{1 + \exp(0.0061\varphi + 0.01505T_{wb} - 1.57084)}$$

(19)

For the ANN models, training and testing results are given in Table 2. As can be seen from this table, errors are within acceptable limits. For the testing data set, the MAPE ranged from 0.01038\% to 0.42516\%.

The maximum MAE was calculated to be 0.01052 °C for $T_{dp}$. Otherwise; the best result was calculated to be 0.00020 kPa for $P_{ws}$.

Figures 1 and 2 present comparisons between analytical data and ANN predictions for the training data set and testing data set, respectively. As observed from the figures, the prediction results agree quite closely with the corresponding analytical data. According to the results derived, the developed ANN models provide a simple and accurate prediction to determine moist air thermodynamic properties in a non-iterative method.

| Table 2. Performance values of ANN models |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | Training data set | Testing data set |
|                | MAE    | MAPE   | MAE    | MAPE   |
| $P_{ws}$ (kPa) | 0.0001 | 0.0092 | 0.0002 | 0.0104 |
| $P_p$ (kPa)    | 0.0001 | 0.0113 | 0.0002 | 0.0126 |
| $T_{dp}$ (°C)  | 0.0037 | 0.0521 | 0.0105 | 0.4252 |
| $T_{wb}$ (°C)  | 0.0058 | 0.0525 | 0.0068 | 0.0614 |
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Figure 1. Comparison between prediction of ANN and analytical results for training data set (a) $P_{ns}$, (b) $P_w$, (c) $T_{dp}$, (d) $T_{wb}$

Figure 2. Comparison between prediction of ANN and analytical results for testing data set (a) $P_{ws}$, (b) $P_w$, (c) $T_{dp}$, (d) $T_{wb}$

Figure 3. The variation of $P_{ws}$ obtained from Eq. (16)
Figure 4. The variation of \( P_{ws} \) obtained from Eq. (17)

\( P_{ws} \) was estimated with 0.01% error and Eq. (16) was obtained for \( P_{ws} \). The variation of \( P_{ws} \) between 5 °C and 30 °C is illustrated in Figure 3. It can be seen from the figures that \( P_{ws} \) value depends only on \( T_{db} \) and it increases linearly with \( T_{db} \). After obtaining the \( P_{ws} \) equation, Eq. (17) was achieved by using the \( P_{w} = \alpha P_{ws} \) equation. \( P_{w} \) can be estimated with approximately 0.01% error with Eq. (17).

The variation of \( P_{ws} \) between 5°C and 30°C is illustrated in Figure 3. It can be seen from the figures that \( P_{ws} \) value depends only on \( T_{db} \) and it increases linearly with \( T_{db} \). After obtaining the \( P_{ws} \) equation, Eq. (17) was achieved by using the \( P_{w} = \alpha P_{ws} \) equation. \( P_{w} \) can be estimated with approximately 0.01% error with Eq. (17). The variation of \( P_{ws} \) between the values 5°C ≤ \( T_{db} \) ≤ 30°C and 55% ≤ \( \phi \) ≤ 80% is presented in Figure 4. As can be seen from the figure, \( P_{w} \) value increases linearly with the values of \( \phi \) and \( T_{db} \). \( T_{dp} \) can be estimated with approximately 0.04% error by using Eq. (18). The variation of \( T_{dp} \) between the values 5°C ≤ \( T_{db} \) ≤ 30°C and 55% ≤ \( \phi \) ≤ 80% is demonstrated in Figure 5. It can be seen from the figure that the value of \( T_{dp} \) increases linearly with the values of \( \phi \) and \( T_{db} \). In order to calculate \( T_{wb} \) mathematically, iteration must be done. But, \( T_{wb} \) can be calculated with approximately 0.4% error by using Eq. (19). The variation of \( T_{wb} \) between the values 5°C ≤ \( T_{db} \) ≤ 30°C and 55% ≤ \( \phi \) ≤ 80% is shown in Figure 6.

4. CONCLUSION

In this study, artificial neural network (ANN) models were presented for the prediction of moist air thermodynamic properties. The new equations were developed to predict \( P_{ws}, P_{w}, T_{dp} \) and \( T_{wb} \) as a function of \( \phi \) and \( T_{db} \). These equations are valid for relative humidities between 55% and 80% and for dry bulb temperatures between 5°C and 30°C. Over the valid range, the obtained equations generally resulted in a good statistical performance with MAPEs in the range of 0.01038–0.42516%. This study reveals that, as an alternative to mathematical models, the moist air thermodynamic properties can be modeled accurately using the ANN approach. The advantage of this approach is that having \( \phi \) and \( T_{db} \) variables, \( P_{ws}, P_{w}, T_{dp} \) and \( T_{wb} \) can be predicted quickly and satisfactorily. This approach can help manufacturers further in order to reduce time and engineering efforts.

5. REFERENCES

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